Estimation of Border-Strip Soil Hydraulic Parameters

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Abstract: The inverse problem of determining soil hydraulic parameters (saturated hydraulic conductivity and water retention parameters) of border-strip irrigation from irrigation event data is analyzed. The inverse problem is solved using sequential unconstrained minimization technique. The forward problem involves the solution of coupled Saint-Venant's equation governing overland flow and Richard's equation governing subsurface flow. Saint-Venant's equations are solved using the MacCormack scheme–based finite-difference method while Richard's equation is solved using a mass conservative fully implicit finite-difference method. Field experiments are conducted on two border strips to obtain surface and subsurface irrigation data such as irrigation advance, recession, flow depth, and soil moisture content. The soil hydraulic parameters, i.e., saturated hydraulic conductivity and soil retention parameters, are estimated by minimizing the deviations between the model-predicted and field-observed irrigation data. The results indicate that defining the objective function in terms of flow depths results in the optimization algorithm converging to the true values as compared to the use of irrigation advance data. Further, it is observed that underestimating the initial guess results in the least number of iterations for the optimization algorithm to converge to the true values. It is also observed that simultaneous estimation of all three soil hydraulic parameters is not possible even with the inclusion of subsurface moisture content data in the objective function. **DOI: 10.1061/(ASCE)IR.1943-4774.0000398.**

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Introduction

Surface irrigation methods such as basin irrigation, border-strip irrigation, and furrow irrigation are the oldest and most commonly used irrigation methods. The four fundamental concerns in the characterization of surface irrigation field properties as outlined by Valiantzas (1994) and Strelkoff et al. (2000) are (1) selection of a suitable mathematical model, (2) estimation of parameters and their spatial and temporal variation, (3) errors in the estimates, and (4) resulting errors in the predicted performance. Numerical models are being widely used for predicting surface irrigation events such as advance, storage, recession, and subsurface wetting. The accuracy of the results from these models heavily depends on the field parameters supplied to these models (Katopodes et al. 1990). Numerous investigations have been conducted attempting either to measure these parameters directly or estimate these parameters from inverse procedures. Even though direct measurements are technically preferable, the parameters obtained may not be applicable at field scale (Elliott et al. 1982). Indirect evaluation of the field parameters by numerical inversion of the governing equations has become an effective alternative to direct methods for the estimation of surface irrigation parameters. In such an approach, a mathematical model is assumed to mimic the physical process and the field parameters are estimated by minimizing the deviation between the model-predicted and field-observed flow attributes such as irrigation advance, recession, and subsurface wetting profile. Contrary to the direct methods, the optimization approach does not put any inherent constraint on the form or complexity of the model, on the stipulation of the initial and boundary conditions, on the constitutive relationships, or on the treatment of inhomogeneities via deterministic or stochastic representations. Thus, a major advantage is that experimental conditions can be selected on the basis of convenience and expeditiousness, rather than by a need to simplify the mathematics of the direct inversion process. The main disadvantage of inverse procedure is that the inverse problem is often ill posed (Carrera and Neumann 1986), which may be due to unidentifiability, nonuniqueness, or instability (Russo et al. 1991). Identifiability refers to the forward relationship. If more than one set of parameters leads to the same response, the parameters are said to be unidentifiable. In such situations, the inverse procedure results in multiple sets of optimal estimates even if the data are free from errors (Ghidaoui and Prasad 2000). Uniqueness refers to the inverse relationship. The inverse solution is nonunique whenever the criterion to be minimized is nonconvex, i.e., it has local minimums at more than one point in the parametric space (Russo et al. 1991) and in situations where the objective function is very flat (Valiantzas and Kerkides 1990). Instability occurs when a small error in the data results in large errors in the parameter estimates (Kool et al. 1987).

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Accurate prediction of surface irrigation events requires the knowledge of system parameters such as Manning's roughness coefficient and soil hydraulic parameters, and saturated hydraulic conductivity and water retention parameters. Numerous studies in the literature have estimated/identified surface irrigation parameters using inverse procedure. Norum and Gray (1970) and Merriam (1985) estimated the parameters of power-law models of a surface irrigation system from irrigation advance. However, the parameters so obtained may not be useful for other models, thus providing lumped estimates of field coefficients as opposed to true parameters describing the field conditions. Katopodes et al. (1990) examined the conditions of observability and parameter identifiability for surface irrigation advance using analytical techniques and the linearized zero-inertia model. The study showed that the linearized zero-inertia model is conditionally observable and the roughness and two infiltration parameters cannot be identified from measurements of the rate of advance alone. However, these three parameters can be identified from measurement of the surface water profile. Katopodes et al. (1990) estimated the surface irrigation parameters by minimizing the deviations between field observations and linearized zero-inertia model predictions. The minimization was carried out using conjugate gradient and variable metric techniques. Katopodes et al. (1990) concluded that formulation of the direct problem and its numerical solution plays a key role in the optimization and the search converges quickly when the influence of independent parameters can be decoupled during construction of the objective function. Bautista and Wallender (1993) studied the identification of furrow infiltration parameters by minimizing the squared difference of observed and model-predicted advance times as a function of distance. They also investigated the identifiability using an alternative objective function in terms of velocity of the advancing wave. The Marquardt algorithm was used in the optimization. The Bautista and Wallender (1993) study indicated that faster convergence and larger radius of convergence is achieved when velocities are used in objective function rather than advance times. Their study also showed that measurement errors and system perturbations impede the identification process. Yost and Katopodes (1998) studied identification of bed resistance and infiltration parameters in surface irrigation based on global procedures. The algorithm was a combination of both global and local procedures exploiting their advantages and did not require an initial guess. Additional enhancements regarding parameter scaling, gradient modification, and combination with the Gauss-Newton method yield a robust and efficient algorithm applicable to a variety of practical problems. Khatri and Smith (2005) compared the performance of six infiltration methods on evaluating the infiltration parameters from irrigation advance data. Their study showed that the two-point method resulted in the best estimates compared to one-point and simpler linear infiltration models. Gillies and Smith (2005) included the runoff data in addition to the advance data in the estimation of infiltration parameters. They concluded that even though the inclusion of runoff data does not improve the parameter estimation to a large extent, accuracy in the estimation of volume of irrigation water has significantly improved. Clemmens and Bautista (2009) studied the applicability of theoretical infiltration equations for estimation of surface irrigation infiltration. They suggested the addition of an offset parameter to the Philip equation to account for cracking and soil consolidation upon wetting. The review of literature suggests that, even though versatile, coupled, surface–subsurface flow models are available for improved basin irrigation management (Zerihun et al. 2005), most of the estimation studies employ surface events such as irrigation advance, recession, and flow depth data in the parameter estimation due to the difficulty in obtaining subsurface flow attributes. The effect of inclusion of subsurface data such as wetting front movement and moisture content on the accuracy of the parameter estimates is not studied in detail (Zerihun et al. 2005). Further, the identifiability and robustness of estimation procedure is also not addressed in much detail. The objectives of the present study are to address (1) the identifiability of the inverse problem, (2) the robustness of the estimation procedure, and (3) the effect of inclusion of subsurface flow attributes on the accuracy of the parameter estimates. For this purpose, a parameter estimation procedure is developed by combining the numerical model simulating border-strip irrigation with an optimization routine based on sequential unconstrained minimization technique (SUMT). The numerical model solves the coupled Saint-Venant's equation governing overland flow on the border strip with Richard's equation governing subsurface flow. Field experiments are conducted on two border strip to collect data on irrigation advance, recession, flow depth, and moisture content in the subsurface. The identifiability issue is addressed by performing parameter estimation of synthetically generated irrigation data. The robustness of the estimation procedure is studied by starting the initial guess values of the parameters one order away from the true values.

Governing Equations

Irrigation events of a border strip irrigation are commonly analyzed by solving coupled differential equations governing overland and subsurface flow on the borderstrip. In the present study, it is assumed that the overland flow is governed by Saint-Venant's equations while the subsurface flow is assumed to be governed by the Richard's equation.

Overland Flow

The partial differential equations governing overland flow in a wide rectangular border strip in Cartesian coordinates (Singh and Bhallamudi 1996) are

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} + q_s = 0 \tag{1}$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{h} + \frac{gh^2}{2}\right) = gh\,(S_0 - S_f) \tag{2}$$

where h = flow depth, q = discharge per unit width, g = acceleration due to gravity, q_s = volumetric rate of infiltration per unit area, S_0 = slope of the border, S_f = friction slope, x = distance along the border strip, and t = time. The friction slope S_f in Eq. (2) is calculated using Manning's formula:

$$S_f = \frac{q^2 n^2}{h^{10/3}} \tag{3}$$

where n = Manning's roughness coefficient. The initial and boundary conditions are described in the following section.

Initial Condition

The values of flow depth and discharge need to be specified as initial conditions. Although dry-bed conditions occur before the start of irrigation in the border, to start the computations, a small uniform initial flow depth is specified at all the nodes at initial time (i.e., $t = 0$) to avoid singularity problems. This is a numerical artifice to predict overland flow. Correspondingly, a uniform discharge computed using Manning's equation is specified as initial discharge throughout the length of the border strip. i.e.,

$$t = 0; \qquad h = h_{\text{ini}}, \qquad q = q_{\text{ini}} \qquad 0 < x < L \tag{4}$$

Boundary Conditions

The boundary conditions at the upstream and downstream end of the border strip depend on the different phases of irrigation such as the advance, storage, and depletion and recession phase. These conditions are described in detail in the following sections.

Advance Phase

The advance phase starts from the instant water is released at the upstream end of the border and continues until the irrigation front reaches the downstream end. The boundary condition is written as

$$0 \leq t \leq t_{ds} \qquad x = 0, \qquad q = q_{us} \tag{5}$$

$$x = L, \qquad q = q_{\text{ini}} \tag{6}$$

where t_{ds} = the time taken for the irrigation front to reach the downstream end and q_{us} = the inlet discharge per unit width.

Storage Phase

The storage phase starts when the irrigation front reaches the downstream end (t_{ds}) and continues up to the instant at which the irrigation supply is cut off. The upstream boundary condition remains the same as in the case of the advance phase, since the discharge q_{us} flows continuously into the border at the upstream end during this phase. The boundary condition is written as

$$t_{ds} < t < t_c; \qquad x = 0, \qquad q = q_{us} \tag{7}$$

where t_c = the water supply cutoff time.

Depletion and Recession Phase

During these phases, the discharge at the upstream end is stopped and, hence, the discharge at the upstream becomes zero. As a consequence, the flow depth at the upstream tends to become zero. It is assumed that recession reaches a point when the flow depth becomes less than or equal to the initial flow depth. Here also, to avoid numerical difficulties, the flow and flow depth are kept as initial flow and flow depth. The boundary is written as follows:

$$h = h_{\text{ini}} \qquad t \geq t_c \qquad x = 0; \qquad q = q_{\text{ini}} \tag{8}$$

However, the boundary condition at the downstream end is kept the same as in the storage phase.

Subsurface Flow

To compute the sink term q_s presented in continuity equation Eq. (1), one needs to know the amount of water infiltrated into the ground. In this study, for the analysis of infiltration process, Richard's equation governing vertical flow in the unsaturated zone is adopted. The mixed form of Richard's equation (Celia et al. 1990) for one-dimensional vertical flow can be written as

$$\frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right)\right] = \frac{\partial \theta}{\partial t} \tag{9}$$

where ψ = the pressure head, θ = the volumetric moisture content, K = hydraulic conductivity, z = the vertical coordinate taken positive upward, and t = the time coordinate. Eq. (9) is nonlinear in nature, because both the flow and storage properties (K and θ) are functions of the dependent variable ψ and its solution requires constitutive relationships for $\theta - \psi$ and $K - \theta$. In the present study, the relationships proposed by Van Genuchten (1980) are adopted for $\theta - \psi$ and $K - \theta$ relationships. The $\theta - \psi$ relationship is described as follows:

$$S_e = \left[\frac{1}{1 + (\alpha_v |\psi|^{n_v})}\right]^{m_v} \quad \text{for } \psi < 0 \qquad S_e = 1 \quad \text{for } \psi \geq 0 \tag{10}$$

where α_v and n_v = unsaturated soil parameters with $m_v = 1 - (1/n_v)$, $n_v > 1$ and S_e = the effective saturation defined as

$$S_e = (\theta - \theta_r)/(\theta_s - \theta_r) \tag{11}$$

where θ_s and θ_r = saturated moisture content and residual moisture content of the soil, respectively.

The $K - \theta$ relationship is described as follows:

$$\begin{aligned} &K = K_s S_e^{1/2}[1 - (1 - S_e^{1/m_v})^{m_v}]^2 \quad \text{for } S_e \leq 1 \\ &K = K_s \quad \text{for } S_e = 1 \end{aligned} \tag{12}$$

where K_s = the saturated hydraulic conductivity of the soil.

For the infiltration, the initial and boundary conditions are described in the following sections.

Initial Condition

Before the start of an irrigation event, the subsurface soil in the border is assumed to be very dry, therefore a very high negative pressure head is assumed as initial condition throughout the thickness of subsurface soil considered:

$$t = 0; \qquad 0 < z < D_s, \qquad \psi = \psi_{\text{ini}} \tag{13}$$

where D_s = the vertical depth of the subsurface considered and ψ_{ini} = the initial pressure head in the subsurface before irrigation.

Top Boundary Condition

The water starts infiltrating into the subsurface soil at a point along the border strip only after the irrigation front reaches that point and continues to infiltrate until the recession front passes through that point. Denoting t_{adv} and t_{rec} as the times at which the irrigation advance and recession fronts arrives at a point, respectively, the top boundary condition is written as

$$t_{\text{adv}} < t < t_{\text{rec}} \qquad z = D_s, \qquad \psi = h \tag{14}$$

where h = the flow depth obtained by solving Eqs. (1) and (2) governing overland flow. At all other times, it is assumed that infiltration is zero at a point.

Bottom Boundary Condition

It is assumed that the water flows freely due to gravity at the bottom of the solution domain. Hence, a gravity drainage boundary condition is specified at the bottom boundary, which is given as follows:

$$t_{\text{adv},} < t < t_{\text{rec}} \qquad z = 0, \qquad \frac{\partial \psi}{\partial z} = 0 \tag{15}$$

Numerical Scheme

Eqs. (1), (2), and (9) are nonlinear partial differential equations and are coupled by the sink term q_s in Eq. (1). These three equations have to be solved simultaneously to obtain the solution. In the present study, MacCormack finite-difference scheme (Singh and Bhallamudi 1996) is used for solving overland flow equations [Eqs. (1) and (2)] and a mass conservative fully implicit finite-difference numerical scheme proposed by Celia et al. (1990) is used for solving Eq. (9).

A finite-difference numerical grid labeled x, z-plane is imposed over the solution domain as shown in Fig. 1. The length of the border strip is divided into uniform segments of length Δx along the x direction. The overland flow equations [Eqs. (1) and (2)] are solved numerically to obtain the flow depth at each surface nodal point. Having obtained the flow depths at each of the surface nodal points, this depth is imposed as the driving head to analyze the moisture flow through the subsurface by solving the Richard's equation [Eq. (9)] along the numerical grid below the surface nodal points. The numerical grid is divided into uniform segments of length Δz along z direction. Having obtained the pressure heads in the subsurface nodal points, the infiltration rate q_s in Eq. (1) is computed at each surface node by applying Darcy's law between the surface node and the node immediately below the surface node, and Eqs. (1) and (2) are solved for q and h. The process of solving overland flow and Richard's equation is continued iteratively until the difference between computed flow depths of two successive iterations falls below a specified tolerance level.

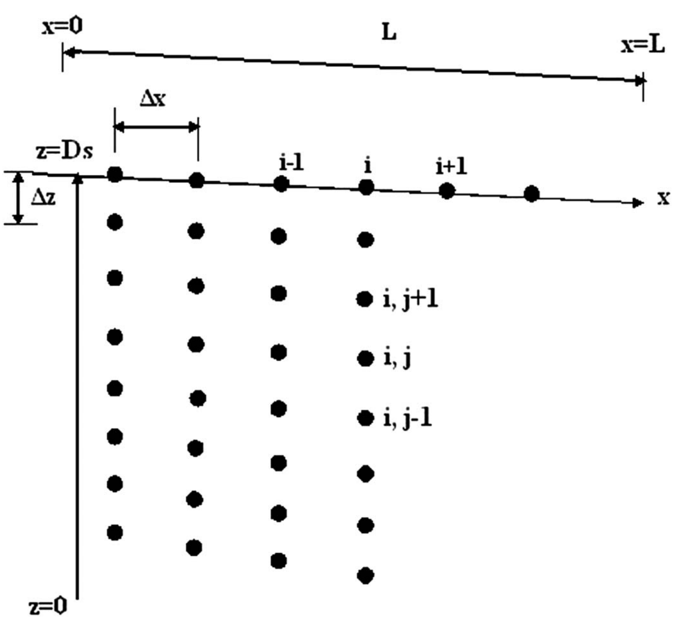

Fig. 1. Finite-difference numerical grid

Inverse Problem

Accurate prediction of border-strip irrigation events requires knowledge of system parameters, Manning's roughness coefficient n, and the following soil hydraulic parameters: saturated hydraulic conductivity $K_{\text{sat}}$and water retention parameters α_v, n_v, θ_s and θ_r. Among these, the estimation of soil hydraulic parameters at the field level is one of the difficult tasks (Walker and Skogerboe 1987) and the present study aims at the estimation of soil hydraulic parameters. For a relatively big field, estimation of these parameters using infiltrometers requires that the infiltration test be conducted at many places. Further, parameters so obtained may not represent the infiltration phenomenon at field scale. An alternative to these direct measurement techniques is to employ inverse techniques for parameter estimation. In such an approach, the soil hydraulic parameters are estimated by minimizing the deviations between the model-predicted and field-observed flow attributes such as irrigation advance, recession, flow depth, wetting front movement, and subsurface moisture contents. The inverse problem involves the estimation of soil hydraulic parameters—K_{sat}, α_v, n_v, θ_s, and θ_r—from irrigation event data of border-strip irrigation. Among these five parameters, θ_s is usually taken as the porosity of the soil and θ_r is a fitting parameter to fit $\theta - \psi$ data at very low moisture content (i.e., very dry state of soil) and does not have much influence on soil moisture dynamics generally encountered during irrigation. Hence, the present study is limited to the estimation of parameters K_{sat}, α_v, and n_v. Table 1 presents the details of the soil hydraulic parameters being estimated using the parameters that are assumed to be constant and the input data for the inverse procedure.

The inverse problem is formulated as a nonlinear optimization problem, i.e., the soil hydraulic parameters are estimated by minimizing the deviation between field-observed and model-predicted responses. The objective function is defined as

$$\min\ \phi(\boldsymbol{b}, L_i) = \sum_{i=1}^{x} v_i \sum_{j=1}^{m_{i,j}} \sum_{k=1}^{m_{i,k}} \sum_{p=1}^{m_{i,p}} W_{j,k,p} [L_i^*(t_j, x_k, z_p) - L_i(t_j, x_k, z_p, \boldsymbol{b})]^2 \tag{16}$$

where L = the different sets of measurements; $m_{i,j}$, $m_{i,k}$, and $m_{i,p}$ = number of specific times, specific distances from the inlet, and specific depths in the subsurface at which measurements are made in a particular set respectively; b = the parameter vector represented as $\boldsymbol{b} = \{K_{\text{sat}}, \alpha_v, n_v\}^T$; L_i^* (t_j, x_k, z_p) = the vector of experimentally observed irrigation advance, flow depth, and moisture contents measured at time t_j, distance x_k, and depth z_p; L_i (t_j, x_k, z_p, $\boldsymbol{b}$) = the vector of model-predicted irrigation advance, flow depth, and moisture contents obtained by solving the direct problem for a given parameter vector $\boldsymbol{b}$; v_i and $\boldsymbol{w}_{j,k,p}$ = the weights associated with a particular measurement set or observation, respectively; $\boldsymbol{w}_{j,k,p}$ is considered as 1; and v_i for each measurement set is taken as the inverse of the measurement variance σ_i^2. The minimization of objective function is accomplished by using sequential unconstrained minimization technique (SUMT). The objective is to find the optimum parameter vector $\boldsymbol{b}$ that minimizes the objective

Table 1. Input Variables, Known and Optimized Parameters of Inverse Procedure

Input variables	Known parameters	Optimized parameters
Irrigation advance	Manning's coefficient n	Saturated hydraulic
Summation of flow depths	Saturated moisture content θ_s	conductivity K_{sat}
Irrigation recession subsurface moisture content	Residual moisture content θ_r	Soil retention parameters α_v and n_v

function. When the observation errors are assumed to be independent and normally distributed, the weighting matrix $\boldsymbol{w}$ becomes an identity matrix and Eq. (1) reduces to a simple ordinary least-squares (OLS) problem

$$\min \phi(\mathbf{b}, L_i) = \sum_{i=1}^{x} v_i \sum_{j=1}^{m_{i,j}} \sum_{k=1}^{m_{i,k}} \sum_{p=1}^{m_{i,p}} [L_i^*(t_j, x_k, z_p) - L_i(t_j, x_k, z_p, \mathbf{b})]^2 \tag{17}$$

The OLS formulation has probably been the most popular one for estimation of parameters. Its appeal is due to its simplicity and the fact that it requires a minimum amount of information. It yields optimal parameter estimates when observation errors are normally distributed, uncorrelated, and have a constant variance (Kool et al. 1987). Table 1 presents the details of input, soil hydraulic parameters considered for estimation, and the parameters that are assumed to be known a priori.

Solution Algorithm for Inverse Problem

The minimization of objective function is carried out using SUMT proposed by Fiacco and McCormick (1968). SUMT is usually a gradient-based traditional optimizer, wherein the problem of constrained minimization is posed as a sequence of unconstrained minimizations by adding sequentially attenuating penalty functions to the objective function. Consider the following typical constrained minimization problem:

Minimize $\phi(\boldsymbol{X})$ with respect to vector $\boldsymbol{X}$, subject to the constraints:

$$g_j(\mathbf{X}) \le 0, j = 1, \ldots, m \tag{18}$$

This problem is converted to the following unconstrained minimization problem:

$$\text{Minimize } \phi(\mathbf{X}) = f(\mathbf{X}) + r \sum_{j=1}^{m} G[g_j(\mathbf{X})] \tag{19}$$

where $G[g_j(\mathbf{X})]$ = the penalty functions associated with violation of the corresponding assigned constraints, and r = an optimization parameter. Typically interior penalty functions $[-1/g_j(\mathbf{X})]$ would be finite and positive in the feasible region, but blow up to plus infinity as the solution approaches the constraint. This is a very strong disincentive for the solution to cross the feasible region, and would thus ensure implementation of the constraints implicitly. However, the solution of the unconstrained minimization would represent the solution of the original constrained minimization problem, provided the penalty parameter r tends to zero. Assigning a very low value to r right in the beginning leads to the problem of poor convergence in the unconstrained minimization of the objective function ϕ. This problem is overcome by initially starting with a moderate value of r, and gradually reducing it through a parameter c in successive unconstrained minimizations, for (k) and $(k+1)$, $(r_{k+1} = c.r_k$ where, $c < 1)$ until the desired level of convergence among sequential unconstrained minimums is obtained. The main advantage of this approach is that one may pick up an appropriate algorithm of unconstrained minimization from a wide array. The direct algorithms for the constrained optimization are fewer and usually far less general. The main problem in this algorithm is the choice of the parameters c and r. Considerable experience and numerical insight is required to make an appropriate choice.

Material and Methods

Most studies on the estimation of surface irrigation parameters employ experimental data consisting of overland measurements such as advance and recession (Walker and Humpherys 1983; Playan et al. 1994; Singh and Bhallamudi 1996). Very few experimental programs involved soil moisture measurements in addition to advance and recession data (Bali and Wallender 1987; Wohling and Mailhol 2007). However, all of these studies discuss the analysis problem (forward problem) and not the parameter estimation problem. The effect of inclusion of subsurface flow data on the identifiability of the soil hydraulic parameters is not addressed at length as most of the studies use analytical expressions such as Kostiakov power law and Parlange's equation for the computation of infiltration (Elliott et al. 1982; Singh and Bhallamudi 1996). The main objective of the experimental program of the present study is to collect the subsurface flow data. Detailed laboratory and irrigation field experiments on two border strips are conducted involving both overland and subsurface measurements. The laboratory and field experiments were conducted at the field experimental station of the Civil Engineering Department, Indian Institute of Technology Roorkee, Uttarakhand, India.

Prior to conducting irrigation experiments at two border strips, various laboratory tests were conducted to determine soil physical characteristics. Soil samples were collected from four locations in each of the border strips, and at each location, two samples were taken from depths of 0–30 cm and 30–60 cm. The grain size analysis of these samples was done using a set of standard sieves. The cumulative particle size curves so obtained were used to determine the fractions of gravel, coarse sand, medium sand, fine sand, silt and clay for each sample. For both border strips, the mean values of gravel, sand, silt, and clay were observed as 0.66%, 74.8%, 17.2%, and 7.4%, respectively. According to the USDA triangular soil classification system, the soil in the two border strips is classified as sandy loam soil. The mean values for bulk density, particle density, and porosity of soil samples were 1.78 g/cm^3, 2.52 g/cm^3, and 0.33, respectively. The saturated moisture content θ_s is taken as the porosity of the soil. The residual moisture content θ_r is determined by fitting the Van Genuchten model with pressure plate extractor data of the soil samples. The average value of θ_r is 0.01. The pressure plate apparatus also provided a representative value of the retention parameters α_v and n_v, which can be used as initial guesses for the inverse problem. The average values of α_v and n_v were obtained as 0.056 cm^{-1} and 1.44, respectively.

Border-Strip Irrigation Experiments

Irrigation experiments were carried out on two border strips in the experimental station. The widths of the two borders are 4 and 5 m, respectively. Prior to the experiment, both border strips were cleared off grass and leveled. The slopes of the two border strips in the experimental station measured with dumpy level were obtained as 0.007 and 0.0106, respectively. The Guelph Permeameter test is carried out at four locations in the border strips and the average value of the saturated hydraulic conductivity is obtained as 2.91 cm/h. Manning's roughness coefficient n is determined using Strickler's formula (Chow 1959) as

$$n = 0.041\, D_{50}^{1/6} \tag{20}$$

where D_{50} = particle size for which 50% of the particles are smaller, in meters. Before conducting the irrigation experiment, stakes were driven into the soil at 1-m intervals to measure the advance and recession of an irrigation event. Tensiometers were installed at

5-m intervals at a depth of 0.4, 0.3, 0.2, and 0.1 m. At locations near the inlet, the sensors were installed at a higher depth and the depth was gradually reduced toward the tail end. The inlet discharges for the two borders were 0.571×10^{-3} and 0.595×10^{-3} $\mathrm{m^3/s/m}$, respectively. Prior to and after conducting the experiment, soil suction and moisture content measurements at depths of every 10 cm up to 60 cm at 5-m intervals along the border strip were taken using tensiometers and a time domain reflectometer. The experimental run was started by irrigating the border strip and measuring the advance, recession, moisture content, and pressure heads. A stop watch was started when water started flowing into the border strip. Advance time and flow depths were recorded as the water reached the successive stakes. Water was allowed to drain freely at the tail end of the border strip. Water inflow into the border strip was continued until cutoff time so that water was infiltrated sufficiently deep into the subsurface to enable soil moisture measurements. Recession measurements were also noted after the cutoff time.

Results and Discussion

The soil hydraulic parameters considered for estimation are the saturated hydraulic conductivity, K_{sat}, and water retention parameters (α_v and n_v). The issues of identifiability and uniqueness are discussed by estimating the parameters from synthetically generated border-strip irrigation event data. The robustness of the optimization procedure is studied by varying the number of unknown parameters (K_{sat}, α_v, and n_v) to be estimated from 1 to 3. In addition, the efficacy of the optimization procedure is analyzed by starting the initial guesses of individual parameters considerably far away from their true values. The parameter estimates are carried out first by giving surface data (irrigation advance and flow depths) to study whether the optimization results in unique estimation of all parameters. In cases where the surface data are found to be inadequate, subsurface data are also included in the optimization. The parameter estimation is discussed in detail in the following sections.

Generation of Synthetic Data

Synthetic irrigation advance, flow depth, and moisture content data were generated by solving the numerical model developed for the simulation of different phases of a border-strip irrigation system to assess the workability of the numerical model. The soil parameters used for generation of synthetic data are $h_{\mathrm{ini}} = 0.005$, $q = 0.0005238\ \mathrm{m^3/s/m}$, $S_0 = 0.007$, $K_{\mathrm{sat}} = 5\ \mathrm{cm/h}$, $n = 0.0305$, $\alpha_v = 0.02\ \mathrm{cm^{-1}}$, $n_v = 2.3$, $\theta_s = 0.33$, and $\theta_r = 0.01$. The length of soil profile is considered 1 m. Prior to irrigation, the soil was assumed to be at an initial pressure head of -1 m. The time of numerical irrigation experiment was 180 min. Irrigation advance, flow depths, and subsurface soil moisture content data at 5-m intervals from the upstream end along the border strip at 10-cm depths until cutoff time were generated. The parameter estimation algorithm was used to estimate the soil hydraulic parameters by employing the synthetically generated data.

Case 1: Estimation of One Unknown Parameter (K_{sat}, α_v, or n_v)

This case considers the estimation of one unknown parameter while treating the other two parameters as constant to their respective values used for the generation of synthetic data. This is an ideal case because inverse procedure is commonly used to estimate more parameters. However, such an analysis may be useful to test the efficacy of the optimization algorithm and to gain an idea of initial guesses of different soil hydraulic parameters fairly close to the minimums while estimating more than one parameter. In case 1, two sub cases (case A and case B) are considered. In case A, the initial guess of the parameter is overestimated, while in case B it is under-estimated. The initial guess of parameters K_{sat} and α_v is over- and underestimated by one order while the range of parameter n_v is considered from 1.2 to 5, as n_v cannot take values less than 1 (Van Genuchten 1980). Table 2 presents the parameter estimates obtained by giving summation of the flow depths and irrigation advance data in the optimization. Table 2 shows that for the case in which the summation of flow depths is used in the optimization, the parameter estimates accurately converge to the true values. However, parameter estimates obtained using irrigation advance data nearly converge to the true values. This is due to the fact that the irrigation advance predicted by the numerical model depends on the grid size (Δx) used for surface nodal points at the ground surface. While predicting the irrigation advance, the numerical model checks whether the flow depth is more than the initial flow depth. For cases where the irrigation advance falls between the surface nodal points, the numerical model assigns the distance to the preceding node as the irrigation advance. Hence, the accuracy of parameter estimates using irrigation advance data can be improved by reducing the surface grid size (Δx). Further, Table 2 suggests that underestimating the initial guess value results in fewer iterations for the optimization to converge to the optimal solution. It is also observed that the provision of both irrigation advance and summation of flow depth data also resulted in estimated parameters converging to the true values. However, using

Table 2. Parameter Estimates for Synthetic Flow Depth and Irrigation Advance Data—Case 1

	Case A (overestimate)				Case B (underestimate)		
	Parameter estimation with flow depth						
Parameter	True value	Initial guess	Final estimate	Number of iterations	Initial guess	Final estimate	Number of Iterations
α_v ($\mathrm{cm^{-1}}$)	0.02	0.2	0.019985	5	0.002	0.019997	3
n_v	2.3	5.0	2.2955	3	1.2	2.2986	2
K_{sat} (cm/h)	5.0	50.0	4.97646	12	0.5	4.98268	9
	Parameter estimation with irrigation advance data						
α_v ($\mathrm{cm^{-1}}$)	0.02	0.2	0.019894	6	0.002	0.019687	5
n_v	2.3	5.0	2.269616	5	1.2	2.272636	4
K_{sat} (cm/h)	5.0	50.0	5.005512	13	0.5	5.00842	8

summation of flow depths in the objective function results in the optimization converging to the true values more rapidly as compared to irrigation advance data. Hence, for further estimation of parameters, the summation of flow depths is used.

Case 2: Estimation of Two Unknown Parameters [(α_v, n_v), (K_{sat}, α_v), and (K_{sat}, n_v,)]

In this case, among the three parameters, two parameters are considered as unknown, keeping the third parameter as constant to its respective value used for generation of synthetic data. Three combinations of two unknown parameters are formed as (α_v, n_v), (K_{sat}, α_v), and (K_{sat}, n_v,). For each of these combinations, four cases are considered. In case A, the initial guess of the parameters is overestimated from their true values. In case B, the initial guess of the parameters is underestimated. In case C, the initial guess of the first parameter is overestimated while the initial guess of the second parameter is underestimated. In contrast, in case D, the initial guess of the first parameter is underestimated, while the initial guess of the second parameter is overestimated. Table 3 presents the details of parameter estimation for cases A, B, C, and D. During the optimization runs, it was observed that when starting the optimization with overestimated values as given in Table 3, the algorithm had problems in converging to the true values. Hence, the overestimated initial guesses are reduced to 20 cm/h, 0.1 cm^{-1}; and 4 for K_{sat}, α_v, and n_v, respectively. It can be seen from Table 3 that the parameter estimates converge nearly to the true values for all cases. In addition, it is also seen that underestimating the initial guess values results in the least number of iterations for the optimization to converge to the true values.

Case 3: Estimation of Three Unknown Parameters (K_{sat}, α_v, and n_v)

This case considers the simultaneous estimation of all three parameters, K_{sat}, α_v, and n_v, from flow depth as well as flow depth and moisture content data. Table 4 shows the details of parameter estimation when flow depth data alone are used in the objective function for optimization. Table 4 shows that the optimization does not converge to the true values for either under - or overestimated initial guesses. This is due to the unidentifiability of the soil hydraulic parameters as discussed by Katopodes et al. (1990). Therefore, in addition to flow depth data, the moisture content data at different depths during irrigation advance is also included in the objective function for optimization. Table 5 shows the parameter estimation details when moisture content data in addition to the flow depth data are used in the optimization. In this case, it was observed that optimization does not converge to the true values for under-, over-, and mixed-estimated initial guesses. A similar observation was also made by Katopodes et al. (1990) during simultaneous estimation of

Table 3. Parameter Estimates for Synthetic Flow Depth Data—Case 2

		Case A (overestimate)			Case B (underestimate)			Case C (mixed) type I			Case D (mixed) type II		
Parameters	True values	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations
α_v (cm^{-1}) n_v	0.02	0.1	0.020092	16	0.002	0.019899	15	0.1	0.02014	21	0.002	0.01954	19
	2.3	4.0	2.27861		1.2	2.28854		1.2	2.28576		4.0	2.28654	
K_{sat} (cm/h)	5.0	20.0	4.971376	20	0.5	4.960213	14	20.0	4.965846	18	0.5	4.89965	22
α_v (cm^{-1})	0.02	0.1	0.01896		0.002	0.01956		0.002	0.01869		0.1	0.01905	
K_{sat} (cm/h)	5.0	20.0	4.89764	18	0.5	4.97902	13	20.0	4.94602	15	0.5	4.970238	17
n_v	2.3	4.0	2.27985		1.2	2.27835		1.2	2.29483		4.0	2.28963	

Table 4. Parameter Estimates for the Synthetic Flow Depth Data—Case 3

		Case A (overestimate)			Case B (underestimate)			Case C (mixed) type I		
Parameters	True values	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations
α_v (cm^{-1})	0.02	0.1	0.021707	17	0.002	0.017905	15	0.1	0.021812	19
n_v	2.3	4.0	2.155833		1.2	2.17654		1.2	2.167542	
K_{sat} (cm/h)	5.0	20.0	5.200005		0.5	4.725563		20.0	5.175846	

Table 5. Parameter Estimates for the Synthetic Flow Depth and Moisture Content Data—Case 3

		Case A (overestimate)			Case B (underestimate)			Case C (mixed) type I		
Parameters	True values	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations	Initial guess	Final estimated value	Number of iterations
α_v (cm^{-1})	0.02	0.1	0.021603	16	0.002	0.01815	14	0.1	0.021926	18
n_v	2.3	4.0	2.37056		1.2	2.20048		1.2	2.40245	
K_{sat} (cm/h)	5.0	20.0	5.30081		0.5	4.82573		20.0	5.30748	

three parameters from surface and subsurface profile depth. It can be concluded that simultaneous identification of all three soil hydraulic parameters is not possible with both flow depth and moisture content data.

Estimation of Soil Hydraulic Parameters from Field Experiments

Having applied the parameter estimation model to synthetic data, the model is used to estimate the soil hydraulic parameters from the two border-strip irrigation experiments. Because the three soil hydraulic parameters (K_{sat}, α_v, and n_v) cannot be identified uniquely from advance, flow depth, and moisture content data, only two parameters, α_v and n_v, are estimated by fixing the value of K_{sat} at 2.92 cm/h which was obtained from the field measurement using the Guelph Permeameter. Table 5 shows the optimal soil hydraulic parameter estimates obtained from border-strip experiments. Figs. 2–4 show the experimentally observed and model-predicted irrigation advance, recession, and moisture content profiles, respectively, using the optimal parameter estimates given in Table 6 for border-strip 1. Similarly, Figs. 5–7 show the experimentally observed and model-predicted irrigation advance, recession, and moisture content profiles, respectively, using the optimal

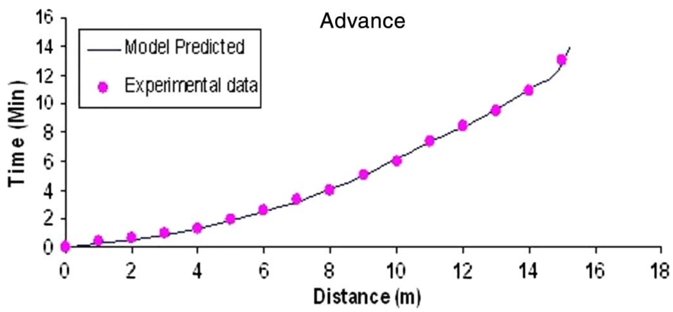

Fig. 2. Comparison of model-predicted and experimentally observed advance for border-strip 1

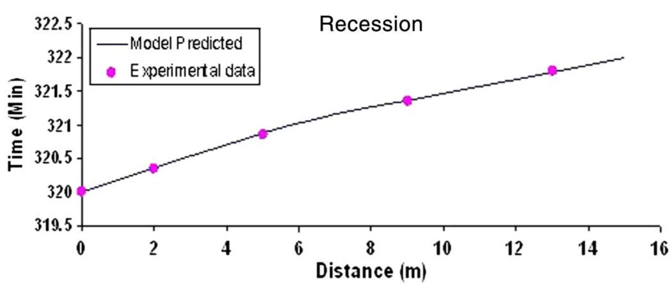

Fig. 3. Comparison of model-predicted and experimentally observed recession for border-strip 1

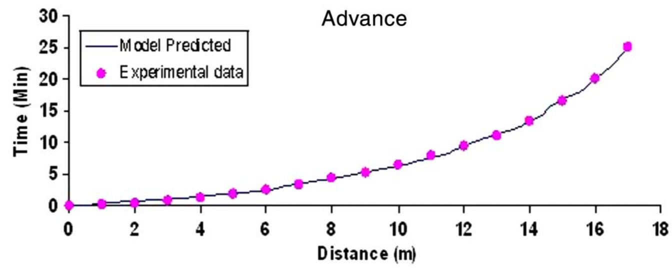

Fig. 5. Comparison of model-predicted and experimentally observed advance for border-strip 2

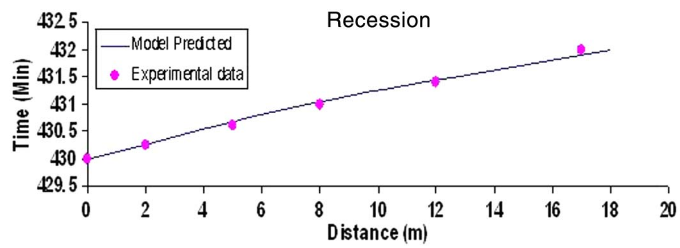

Fig. 6. Comparison of model-predicted and experimentally observed recession for border-strip 2

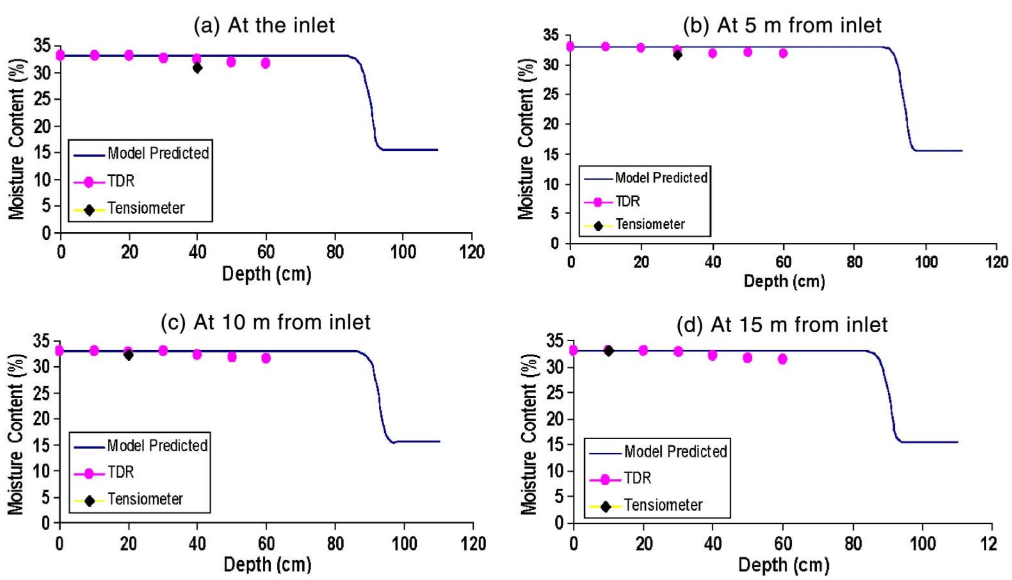

Fig. 4. Comparison of model-predicted and experimentally observed moisture content profile for border-strip 1 (at cutoff time)

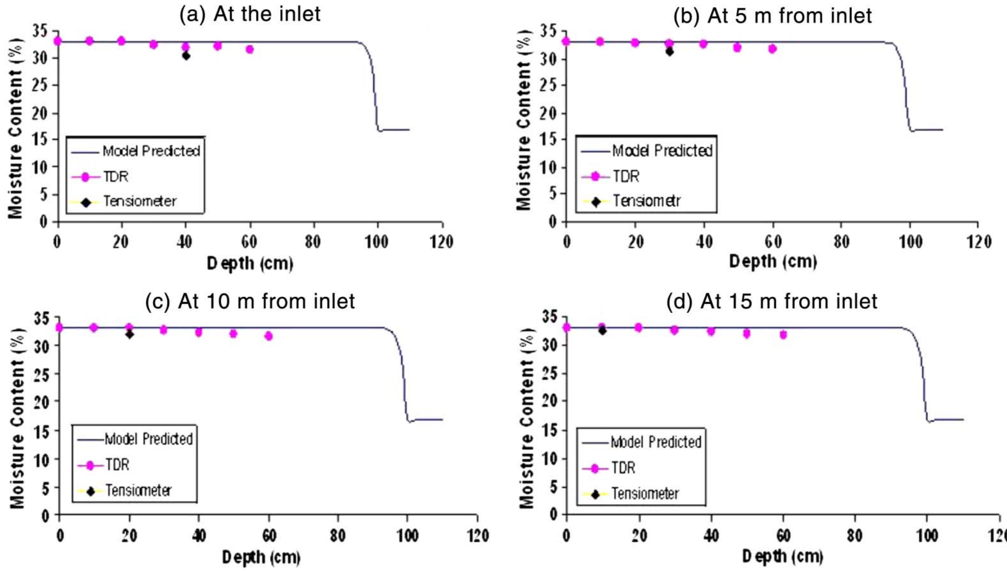

Fig. 7. Comparison of model-predicted and experimentally observed moisture content profile for border-strip 2 (at cutoff time)

Table 6. Optimal Infiltration Parameter Estimates from Border-Strip Experiments

Parameters	Border-strip 1	Border-strip 2
α_v (cm^{-1})	0.054	0.05
n_v	1.45	1.42

parameter estimates given in Table 6 for border-strip 2. It can be seen from Figs. 2–7 that the model-predicted irrigation advance, recession, and moisture contents using the optimal parameter estimates are in close agreement with the field observations in both border strips. This indicates that the inverse procedure can be employed with confidence in the estimation of two of the three border-strip soil hydraulic parameters.

Conclusions

The present study is concerned with the estimation of soil hydraulic parameters from border-strip irrigation event data. The parameter estimation problem is formulated as a least-squares minimization problem wherein the parameters are estimated by minimizing the deviations between the field-observed and model- predicted irrigation events such as advance, flow depths, and moisture contents. An optimization model is developed by coupling sequential constrained minimization technique with a numerical model predicting both overland and subsurface flows of border-strip irrigation. The numerical model employs the MacCormack scheme–based finite-difference scheme for overland flow and a mass conservative fully implicit scheme for the subsurface flow. Irrigation experiments are conducted at two border strips.

It is found that with only irrigation advance and summation of flow depths data, only two of the three parameters (K_{sat}, α_v, and n_v) can be uniquely estimated. During optimization, it is observed that defining the objective function in terms of flow depth results in the optimization algorithm converging to the true values of the parameters as compared to irrigation advance. For the case of estimation of two parameters, underestimating the initial guess values results in the least number of iterations for the optimization algorithm to converge to the true values. Inclusion of moisture content data in the objective function does not ensure unique estimation of all three parameters. Parameter estimation using experimental data of two border-strip experiments indicates that the parameter estimates are quite close to the values obtained using direct measurements. This indicates that parameter estimation technique can be applied with confidence for the estimation of soil hydraulic parameters.

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